Conformal Viscous Hydrodynamics

P. Romatschke

INT, University of Washington, Seattle

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Outline

Motivation for Viscous Hydrodynamics

Usually I give a long introduction here...

...but you're all experts!

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Why conformal hydro?

- I'm interested in the effects of *shear viscosity* η
- There's also bulk viscosity ζ, which comes from

$$\zeta \sim T^{\mu}_{\mu}$$

- Ignoring effects from ζ: set $\zeta = 0$. Implies

$$T^{\mu}_{\mu} = 0$$

- Conformal invariance!

Conformal Viscous Hydro

- Baier, PR, Son, Starinets, Stephanov, arXiv:0712.2451:

$$\begin{aligned}\Pi^{\mu\nu} &= \eta\nabla^{\langle\mu}u^{\nu\rangle} - \tau_\Pi\left[\Delta^\mu_\alpha\Delta^\nu_\beta D\Pi^{\alpha\beta} + \frac{4}{3}\Pi^{\mu\nu}(\nabla_\alpha u^\alpha)\right] \\ &\quad + \frac{\kappa}{2}\left[R^{<\mu\nu>} + 2u_\alpha R^{\alpha<\mu\nu>\beta}u_\beta\right] \\ &\quad - \frac{\lambda_1}{2\eta^2}\Pi^{<\mu}{}_\lambda\Pi^{\nu>\lambda} + \frac{\lambda_2}{2\eta}\Pi^{<\mu}{}_\lambda\omega^{\nu>\lambda} - \frac{\lambda_3}{2}\omega^{<\mu}{}_\lambda\omega^{\nu>\lambda}\end{aligned}$$

- Invariant under conformal transformations $g_{\mu\nu} \to e^{-2\omega}g_{\mu\nu}$
- Most general conformal expression to 2nd order in gradients
- Five 2nd order coefficients $\tau_\Pi, \kappa, \lambda_1, \lambda_2, \lambda_3$ can be matched to weak coupling (Boltzmann) or strong coupling ($\mathcal{N} = 4$ SYM) plasmas

Conformal Viscous Hydro vs full Israel-Stewart

$$
\begin{aligned}
\boxed{\Pi^{\mu\nu}} \quad = \quad & \boxed{\eta\nabla^{\langle\mu}u^{\nu\rangle}} - \boxed{\tau_\Pi\left[\Delta^\mu_\alpha\Delta^\nu_\beta D\Pi^{\alpha\beta} + \tfrac{4}{3}\Pi^{\mu\nu}(\nabla_\alpha u^\alpha)\right]} \\
& + \frac{\kappa}{2}\left[R^{<\mu\nu>} + 2u_\alpha R^{\alpha<\mu\nu>\beta}u_\beta\right] \\
& - \frac{\lambda_1}{2\eta^2}\Pi^{<\mu}{}_\lambda\Pi^{\nu>\lambda} + \boxed{\tfrac{\lambda_2}{2\eta}\Pi^{<\mu}{}_\lambda\omega^{\nu>\lambda}} - \frac{\lambda_3}{2}\omega^{<\mu}{}_\lambda\omega^{\nu>\lambda}
\end{aligned}
$$

- Only one 2nd order coefficient: τ_Π $(\lambda_2 = -2\eta\tau_\Pi)$
- Cannot be matched to strongly coupled theories ($\mathcal{N} = 4$ SYM)

Conformal Viscous Hydro vs full Israel-Stewart

- Both have finite propagation speeds

$$v_{\rm max} = \sqrt{\frac{\eta}{\tau_\Pi(\epsilon+p)}}$$

- Both have $v_{\rm max} < 1$ for weak coupling
- Conformal hydro for strong coupling ($\mathcal{N} = 4$ SYM) also has $v_{\rm max} < 1$:

$$\tau_\Pi = \frac{2(2-\ln 2)\eta}{\epsilon+p}, \; \kappa = \frac{\eta}{\pi T}, \; \lambda_1 = \frac{\eta}{2\pi T}, \; \lambda_2 = -\frac{\eta \ln 2}{\pi T}, \; \lambda_3 = 0$$

BRSSS07, Bhattacharyya e.a. arXiv:0712.2456,
Natsuume & Okamura arXiv:0712.2916

Why can IS not be matched to strong coupling? (1/2)

Calculate Green's function for tensor metric perturbation $\delta g_{xy}(t,z)$ and sound dispersion in hydro (BRSSS)

$$
\begin{aligned}
G_R^{xy,xy} &= p - i\eta\omega + \eta\tau_\Pi\omega^2 - \frac{\kappa}{2}\left[\omega^2 + k^2\right] + \ldots, \\
\omega &= c_s k - i\Gamma k^2 + \frac{\Gamma}{c_s}\left(c_s^2\tau_\Pi - \frac{\Gamma}{2}\right)k^3 + \ldots,
\end{aligned}
$$

where $\Gamma = \frac{2\eta}{3sT}$. IS amounts to $\kappa \equiv 0$.

Why can IS not be matched to strong coupling? (2/2)

Calculate Green's function for tensor metric perturbation $\delta g_{xy}(t,z)$ and sound dispersion using AdS/CFT:

$$\begin{aligned} G_R^{xy,xy} &= \frac{\pi^2 N_c^2 T^4}{8} - \frac{\pi N_c^2 T^3}{8} i\omega - \frac{N_c^2 T^2}{16}\left[-\omega^2 + k^2 + \omega^2 \ln 2\right] + \ldots, \\ \omega &= \frac{1}{\sqrt{3}}k - \frac{i}{6\pi T}k^2 + \frac{3 - 2\ln 2}{24\pi^2\sqrt{3}T^2}k^3 + \ldots. \end{aligned}$$

Consistency *requires* $\kappa \neq 0$. IS is not general enough!

Where does this mismatch come from?

- Differences between IS and BRSSS show up only at 2nd order in gradients
- One way to derive IS is from Boltzmann equation. Boltzmann equation is itself a gradient expansion (to first order) of underlying QFT. 2nd order beyond accuracy of coarse-graining!
- Another way to derive IS is from requiring $\partial_\mu s^\mu \geq 0$. IS require positivity for arbitrarily strong gradients (high momenta). Hydrodynamics: 2nd order always small compared to 1st order, positivity guaranteed.

Conformal Hydro and Heavy-Ion Collisions

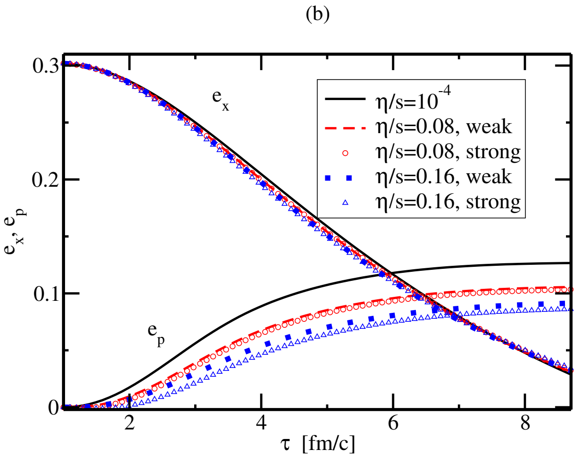
- Most general, causal, relativistic conformal hydro has five 2nd order transport coefficients $\tau_\Pi, \kappa, \lambda_1, \lambda_2, \lambda_3$
- κ multiplies Ricci and Riemann tensor: not needed in flat space
- λ_2, λ_3 multiply vorticity tensor: for boost-invariant hydro, dynamics is only in transverse plane (2d). Can derive relativistic vorticity equation (PR+UR, arXiv:0706.1522)

$$D\omega^{xy} + \omega^{xy}\left[\nabla_\mu u^\mu + \frac{Dp}{\epsilon + p} - \frac{Du^\tau}{u^\tau}\right] = \mathcal{O}(\Pi^3).$$

For HIC, term in []'s is usually positiv, so $\omega^{xy} = 0$ is a stable fix point of relativistic (ideal) hydro. Do not expect ω^{xy} to be large for viscous hydro, so λ_2, λ_3 are not needed.

Conformal Hydro and Heavy-Ion Collisions

Dependence on τ_Π, λ_1 (from M. Luzum+PR, 0804.4015)



Weak: $\tau_\Pi = 6\frac{\eta}{sT}, \lambda_1 = 0$; Strong: $\tau_\Pi = 1.3\frac{\eta}{sT}, \lambda_1 = \frac{\eta}{2\pi T}$.

Conformal Hydro and Heavy-Ion Collisions – Summary

- 2nd order conformal hydro theory is clean
- 2nd order conformal hydro is useful for HIC because evolution depends effectively only on one parameter: viscosity
- But extracting η/s from experiment is a mess!

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Things to know about Hydro @ RHIC

For any hydrodynamic model of a heavy-ion collision

- Hydrodynamics = differential equations. Need to fix initial/boundary conditions!
- the time when to start the hydrodynamic evolution
- the initial distribution of energy density (Glauber? CGC?)
- the equation of state for QCD (lattice!)
- the freeze-out procedure (Cooper-Frye?)
- There is much more to RHIC hydro than just fluid dynamics!

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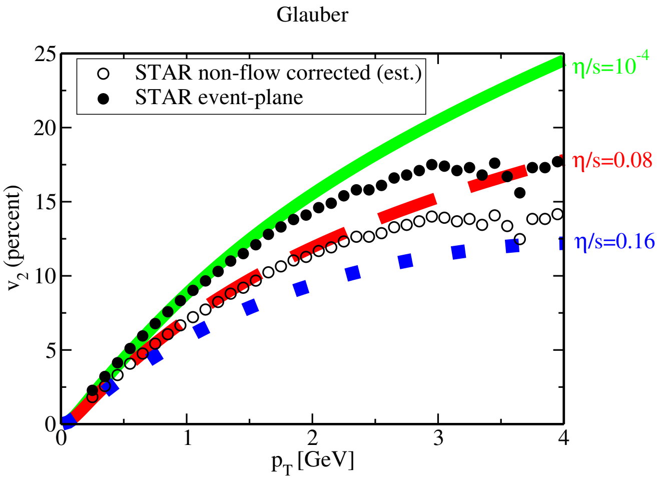
Elliptic flow (min.bias)



PR+UR, PRL99, M. Luzum+PR,arXiv0804.4015

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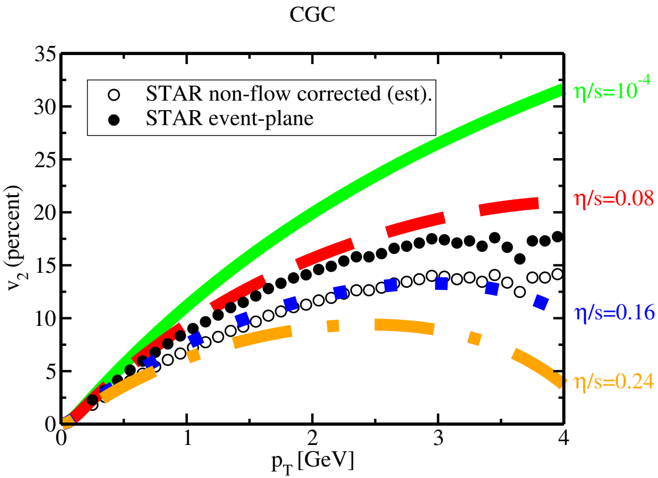

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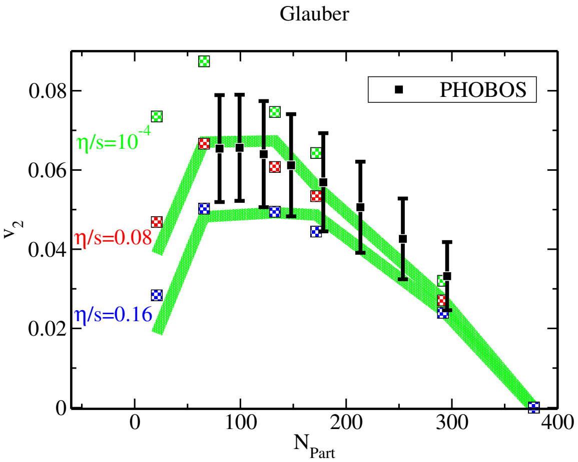
Elliptic flow (integrated)



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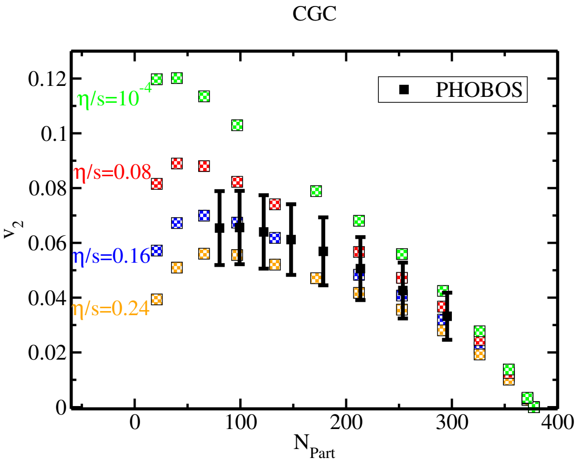

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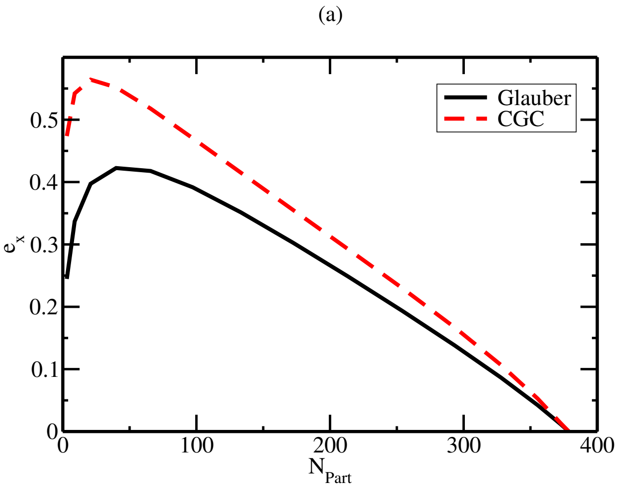
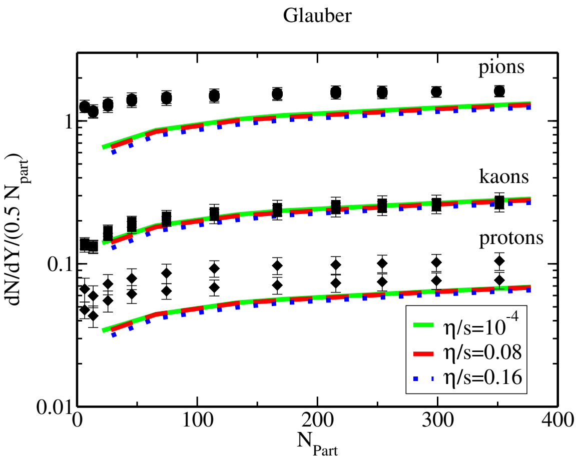
Eccentricity: Glauber vs CGC



CGC a la Drescher, Dumitru, Hayashigaki, Nara

Multiplicity (Glauber)



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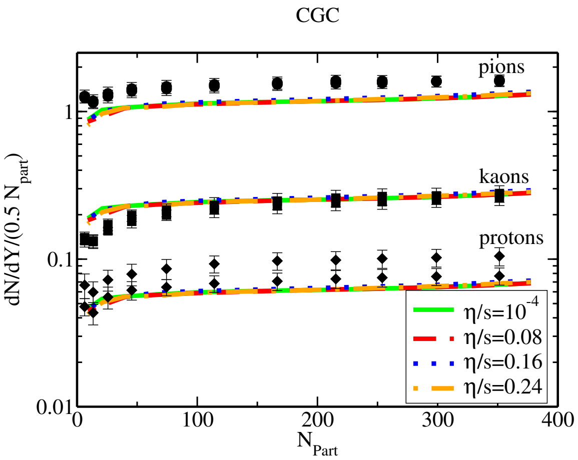
Multiplicity (CGC)



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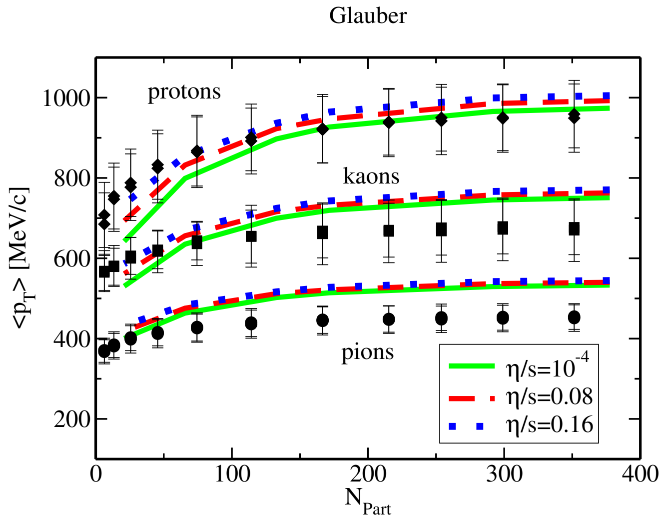
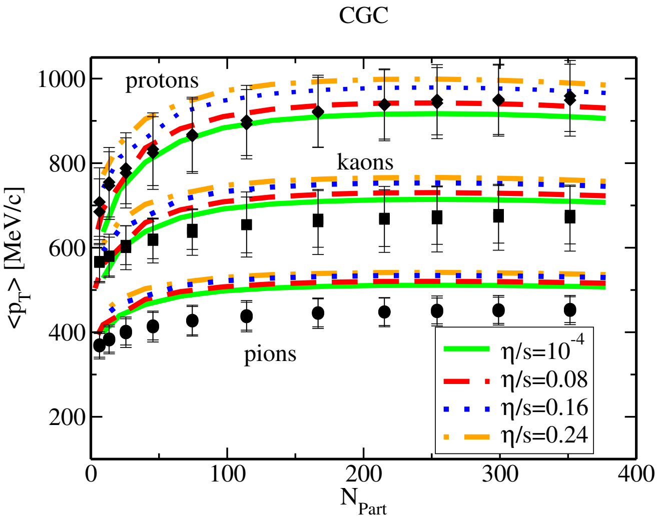
Mean transverse momentum (Glauber)



M. Luzum+PR,arXiv0804.4015

Mean transverse momentum (CGC)



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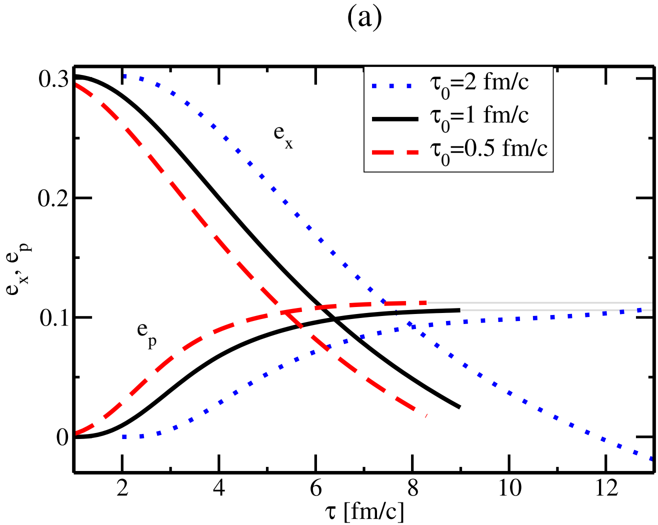
Early Thermalization



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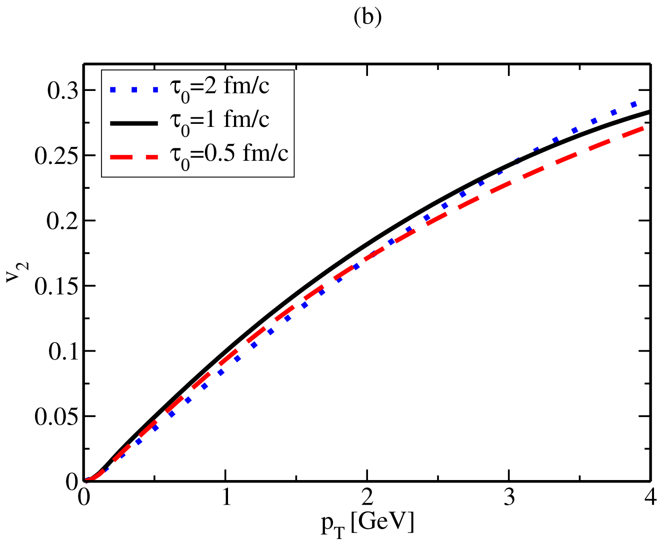

Summary: Status of η/s at RHIC

- Our hydrodynamic model seems to match RHIC data for $\eta/s \sim 0.1 \pm 0.1(\text{theory}) \pm 0.08(\text{experiment})$
- Biggest theory uncertainty from unknown initial state
- Significant uncertainty from experiment (non-flow!)
- With (non-flow corrected) data, KSS bound is consistent with RHIC data, for both Glauber and CGC

To check KSS bound at RHIC, need better data& better hydro!

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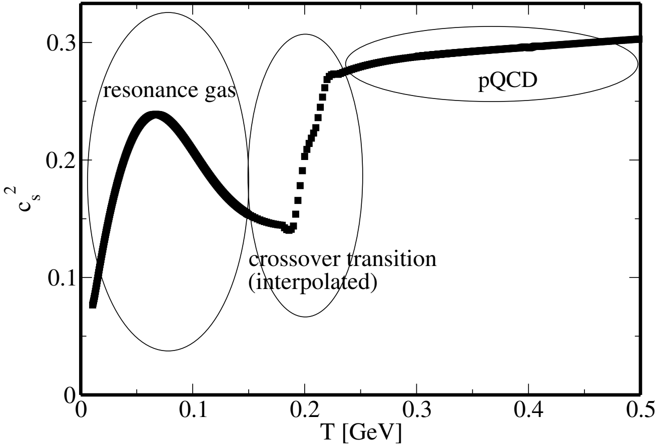
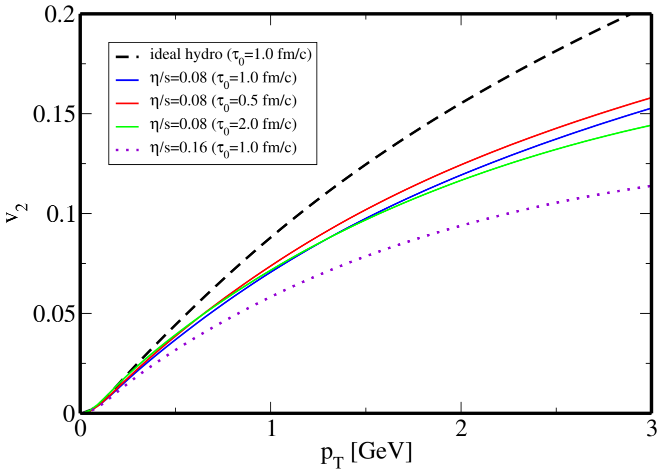
Backup slides

Speed of Sound from Laine and Schröder, PRD73



Dependence on τ_0



Backup: Multiplicity in Viscous Hydro

	$\frac{dN_{\pi,\mathrm{visc}}}{dy} / \frac{dN_{\pi,\mathrm{ideal}}}{dy}$	$\frac{dN_{K,\mathrm{visc}}}{dy} / \frac{dN_{K,\mathrm{ideal}}}{dy}$
$\eta/s = 0.08$	1.06	1.06
$\eta/s = 0.16$	1.12	1.12
$\eta/s = 0.24$	1.18	1.19
$\eta/s = 0.32$	1.23	1.23
$\eta/s = 0.40$	1.28	1.28

Viscous Hydro creates $\sim 0.75\,\eta/s$ more final multiplicity!

Early Thermalization